ISI – Bangalore Center – B Math - Physics III – End Term Exam
Date: 13 November 2017. Duration of Exam: 3 hours
Total marks: 80
Answer Questions 1 to 4 and either Question 5 or Question 6.

Q1. [Total Marks: 3+3+3+2+4=15]

1a.) A linear dielectric material is characterized by a dipole moment per unit volume, also called polarization, $\vec{P}$ which depends on the electric field in the following way $\vec{P} = \varepsilon_0 \chi_e \vec{E}$ where χ_e is the electric susceptibility. Show that $\rho_b = -\left(\frac{\chi_e}{1+\chi_e}\right)\rho_f$ where the symbols have their usual meanings.
1b.) A solid uniform metal sphere of radius *a* carries a total charge *Q*. It is surrounded by a spherical shell uniformly filled with a linear nonconducting dielectric with permittivity ε. The shell with the dielectric material has inner radius *a* and outer radius 1*b*.) Show that the polarization of the dielectric shell is given by $\vec{P} = \frac{\varepsilon_0 \chi_e Q}{4\pi\varepsilon r^2}\hat{r}$
1c.) Calculate the bound charge density ρ_b for the system in part b.). Show that it agrees with the result found in part a.)
1d.) Calculate the surface charge density σ_b associated with the dielectric in part b.)

Q2. [Total Marks: 4+6+4=14]

2a.) Suppose that a linear non-conducting dielectric liquid occupies the space z<0 and that an uncharged metallic sphere is floating on the liquid with 1/5 of its volume submerged. Suppose that sphere is then charged with charge Q and it continues to float on the liquid. Will the charged sphere displace more than, equal to, or less than 1/5 of its volume (neglect the mass of the charges)? Explain your answer preferably with the help of a diagram.

2b.) Consider a conducting material where the current density *j* is proportional to the electric field. If the current is steady so that the local charge density is constant is time, show that the electric field in the conductor obeys the same equations as those of electrostatics, namely $\vec{\nabla}.\vec{E} = 0, \vec{\nabla}\times\vec{E} = 0$.

2c.) The electromagnetic energy density is defined by
$u_{em} = \frac{1}{2}(\varepsilon_0 |\vec{E}|^2 + \frac{1}{\mu_0}|\vec{B}|^2)$ and is in general time dependent even in an area free of charges. Explain how the change in energy density is accounted for in classical electromagnetism. In particular, state without derivation the equation that relates $\frac{\partial u_{em}}{\partial t}$ to a local quantity constructed out of the electric and magnetic fields (and their derivatives) which accounts for energy flow.

Q3.[Total Marks: 5+5+5=15]

A rectangular loop of conducting wire of dimensions l and w is initially held at rest in a region with zero magnetic field. It is released at time t=0 from just above a region with a constant magnetic field of magnitude B as shown in the picture. The loop has resistance R, self inductance L and mass m. Consider the motion of the loop as long as the upper edge remains in the zero magnetic field region.

3a.) Show that the speed of the loop satisfies the equation

$$mL\frac{d^2v}{dt^2} + mR\frac{dv}{dt} + B^2l^2v = mgR$$

3b.) Find the expression for the current if L is so small that it can be neglected.

3c.) Find the expression for the current if R can be so small that it can be neglected.

Please make sure your answer for b.) and c.) are consistent with the initial conditions.

Q4. [Total Marks: 4+4+4+6 =18]

4a.) Write down the Maxwell equations for the electric and magnetic fields in a homogeneous and isotropic dielectric medium (characterized by ε, μ) containing free charge and current density ρ_f, j_f .

4b.)Show that in the absence of a free charges these imply wave equation for E and B fields inside the dielectric medium . What is the speed of propagation of the wave?

4c.) Derive the continuity equations for the electric fields across two dielectric materials.

4d.) Consider a plane wave incident normally on a plane surface separating two dielectric material. Assume that the reflected and refracted wave also are normal to the surface. Show that the boundary condition ensures that the frequency of the incident , reflected and the refracted waves must be same.

Q5. [Total Marks: 6+6+6=18]

The magnetic field in a magnetostatic system is given by the two equations:
$\vec{\nabla}.\vec{B}=0, \vec{\nabla}\times\vec{B}=\mu_0\vec{j}$.

5a.) Show that $\vec{B}$ can be written as the curl of a vector potential $\vec{A}$ which can be chosen in such a way that it satisfies the equation $\nabla^2\vec{A}=-\mu_0\vec{j}$.

5b.) The formal solution of vector potential $\vec{A}$ is given by $\vec{A}(\vec{r})=\frac{\mu_0}{4\pi}\int\frac{\vec{j}(\vec{r}')}{|\vec{r}-\vec{r}'|}d^3r'$.
Using this formal solution, prove that the magnetic field of an infinite solenoid whose central axis is the same as the z axis is given by $\vec{B}(s,\varphi,z)=B(s)\hat{z}$ in the usual cylindrical coordinates.

5c.) Using the result in part b.) and appropriate Amperian loops show that the magnetic field outside an infinite cylinder is zero and that it is constant inside the solenoid. (no need to calculate the field inside)

Q6. [Total Marks: 4+4+5+5=18]

6a.)Starting from the expression of force on a charge particle in an electromagnetic field $\vec{F}=q(\vec{E}+\vec{v}\times\vec{B})$, determine the transformation properties of the electric field and magnetic fields under space inversion. In other words express the relationship between the electric and magnetic fields $\vec{E}'(x',y',z')$, $\vec{B}'(x',y',z')$ in terms of $\vec{E}(x,y,z)$, $\vec{B}(x,y,z)$ respectively where $x'=-x, y'=-y, z'=-z$.

6b.) What are the corresponding transformation properties for these fields under the transformation $x'=x, y'=y, z'=-z$?

6c.) Using only these transformation properties and symmetry arguments, show that the magnetic fields due to a steady current I flowing along an infinite wire of finite uniform cross-section a (the current density is given by $\vec{j}(s,\theta,z)=\frac{I}{\pi a^2}\hat{z}, s\leq a$ and zero otherwise) must be of the form: $\vec{B}(s,\phi,z)=B(s)\hat{\phi}$.

Note that credit will be given for correct approach and for correctly deriving parts of this result.

[Hint: Consider reflection in the xz plane and show that reflection properties of the magnetic field implies that magnetic field at all points on the xz plane must point in the plus or minus the y direction. Use the fact that x axis can be arbitrarily chosen.]

6d.) Find the function $B(s)$ using Ampere’s law.

Figure for Q4.